Cellular automaton model for bidirectional traffic

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We investigate a cellular automaton (CA) model of traffic on a bidirectional two-lane road. Our model is an extension of the one-lane CA model of Nagel and Schreckenberg [J. Phys. I (France) **2**, 2221 (1992)], modified to account for interactions mediated by passing, and for a distribution of vehicle speeds. We chose values for the various parameters to approximate the behavior of real traffic. The density-flow diagram for the bidirectional model is compared to that of a one-lane model, showing the interaction of the two lanes. Results were also compared to experimental data, showing close agreement. This model helps bridge the gap between simplified cellular automata models and the complexity of real-world traffic. [S1063-651X(98)00401-2]

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I. INTRODUCTION

A variety of cellular automaton (CA) models give a discrete approximation to traffic flow. Nagel and Schreckenberg's [1] one-lane model accurately describes some of the main features of real traffic. This led to multilane [2] and two-dimensional (e.g., [3,4]) CA models, which extended the range of traffic phenomena that can be treated by cellular automata. This paper continues in this line by introducing a CA model in which vehicles move on two lanes, in opposite directions. We find that even complicated interactions, such as occur during passing into oncoming traffic, can be represented in a CA model. This supports the view that cellular automata provide a real alternative to differential equations for the modeling of traffic flow.

Multilane models provide rules for lane changes, and we observe nontrivial interactions between the lanes. With two lanes in opposite directions, these interactions depend strongly on the relationship between the densities on the two lanes. When either or both of the densities are large, interactions can be ignored since passing is impossible; our model is equivalent to two copies of the standard one-lane model. If there are no vehicles in the oncoming lane, then our model behaves like an asymmetric model with two lanes in the same direction. If there are only a few vehicles in a given lane and an intermediate number in the oncoming lane, then vehicles in the oncoming lane will pass, and slow progress on the given lane.

The one-lane CA model of [1] assumes that all vehicles have the same maximum velocity. Indeed, even a multi-velocity model quickly settles down to a state where all vehicles move with the lowest speed; the model unrealistically creates platoons of vehicles, each following a slow car. As will be seen below, a distribution of speeds is especially important for realism in the bi-directional model.

II. EXISTING MODELS

a. One-lane model. In the one-lane CA model of [1] each site may be empty or occupied by a car with an integer velocity $v \in \{0, \ldots, v_{\max}\}$. $v_{\max}=5$ or greater gives good agreement with physical experiments. The variable gap gives the number of unoccupied sites in front of a vehicle. p_{decel} is the probability to randomly decelerate, and rand is a random number between 0 and 1. One iteration consists of the three following sequential steps, which are applied in parallel to all cars:

(1) Acceleration of free vehicles: IF ($v < v_{\max}$) THEN $v = v + 1$.

(2) Slowing down due to other cars: IF ($v > \text{gap}$) THEN $v = \text{gap}$.

(3) Stochastic driver behavior: IF ($v > 0$) AND (rand $< p_{\text{decel}}$) THEN $v = v - 1$. These simple conditions already give realistic results.

b. Two-lane unidirectional model. The two-lane unidirectional model is built from two parallel single-lane models [2]. Four additional conditions govern the exchange of vehicles between lanes. First the vehicles change lanes, then the one-lane algorithm is applied. This model introduces several new variables: $\text{gap}_{\text{same}}, \text{gap}_{\text{opp}}$: the number of unoccupied sites in front of vehicle i on the (same, opposite) lane, respectively. $\text{gap}_{\text{behind}}$ the number of unoccupied sites *behind* the vehicle, on the opposite lane; l_{same}, l_{opp}, l_{back}: the minimum free distance needed for a pass, ahead on same lane, ahead and behind on the opposite lane; p_{change}: the probability to change lanes. The added rule is as follows:

(4) IF [($\text{gap}_{\text{same}} < l_{\text{same}}$) AND ($\text{gap}_{\text{opp}} > l_{\text{opp}}$) AND ($\text{gap}_{\text{behind}} > l_{\text{back}}$) AND (rand $< p_{\text{change}}$)] THEN change lane.

This model has a symmetric and an asymmetric version. In the asymmetric version, there is no passing on the right.

III. TWO-LANE BIDIRECTIONAL MODEL

The bidirectional model has several types. Either passing is allowed on both lanes, only one or the other of the lanes, or no passing is allowed. Mixtures of these types are possible; one type may be applied on a given section of roadway, and another type on an adjacent section.

Consider a type in which passing is allowed. An algorithm that controls passing must account for a number of

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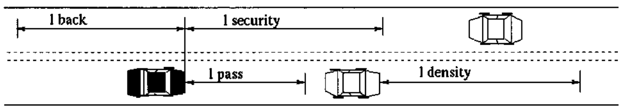

FIG. 1. Bidirectional model.

circumstances. First, a vehicle must not decelerate while passing; $p=0$ during the pass. Moreover, if an oncoming car is seen, the passing car must immediately return to its own lane. In reality vehicles generally do not attempt to pass unless the pass can be completed; a model should reflect this. We let each vehicle measure the ''local density:'' the density of cars in front of the vehicle it would like to pass. If the local density is sufficiently low, the vehicle has a good chance of completing a pass, and we allow it to try. Few passes will occur when the global density is high, even if the density is low on the oncoming lane. The lanes become effectively decoupled.

Three kinds of traffic jams can appear on a bidirectional road. The most common is a start-stop wave on one of the two lanes. Rarely, a jam is caused when an audacious driver tries to pass though there is no space to return to the home lane. Finally, a jam may occur when each of an adjacent pair of drivers, one on each lane, tries to pass simultaneously. Unless the symmetry between lanes is broken (see below), this ''super jam'' halts all traffic on both lanes.

Our CA operates on a lattice with two lanes in opposite directions (see Fig. 1). Each site has a state $v \in \{-(v_{\max}+1) \cdots v_{\max}+1\}$, where 0 represents the absence of a vehicle, ± 1 a stopped vehicle, ± 2 a vehicle moving with speed 1 in the positive (negative) direction and so on.

Vehicle movement is calculated in a two-step process,

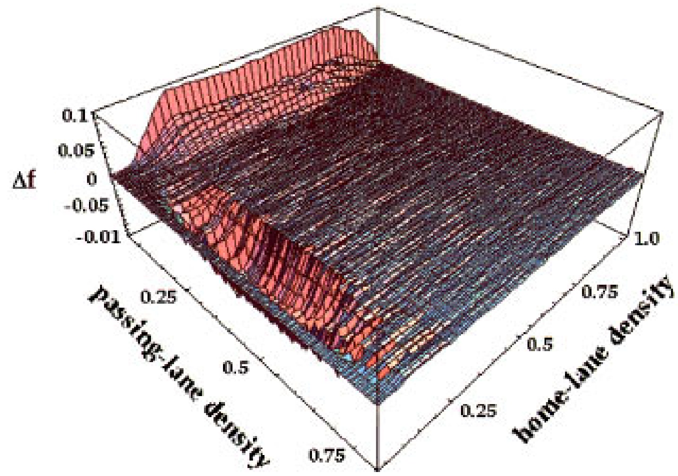

FIG. 3. (Color) Three-dimensional plot of flow on the home lane. The height of the surface gives the difference between flow on the home lane in the bidirectional model and flow in a one-lane model as a function of the densities on the home and passing lanes of the bi-directional model.

following [2]. First vehicles change lanes, then they advance.

In addition to the functions and variables introduced above, we will need the following: $v_{\text{same}}, v_{\text{opp}}$: the velocity of the vehicle ahead on the (same, opposite) lane. H: true iff the vehicle is on its home lane; *oncoming*: true iff $\text{sgn}(v_{\text{same}}) \neq \text{sgn}(v)$; l_{pass}: if $\text{gap}_{\text{same}} < l_{\text{pass}}$ AND H then a pass may be attempted. l_{back}: the distance a driver looks back for obstacles on the passing lane. l_{security}: if $\text{gap}_{\text{same}} < l_{\text{security}}$ AND not(H) then the vehicle returns immediately to its home lane. D_L: local density: the fraction of the $l_{\text{density}} = 2 \times v_{\max} + 1$ sites in front of the given vehicle that

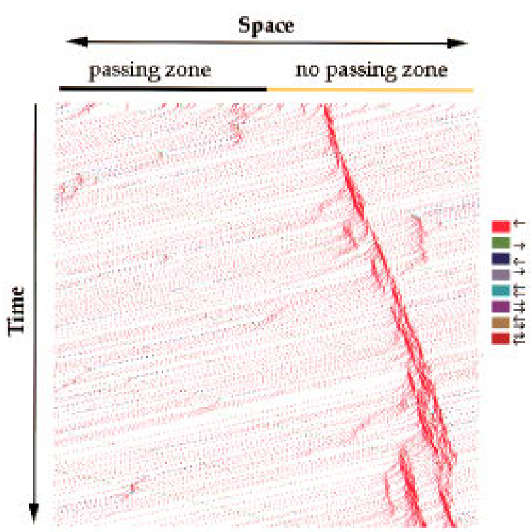

FIG. 2. (Color) Passing and no-passing zones. A cell-wide section of roadway containing only one vehicle moving rightward (leftward) on its home lane is labeled red (green). Other colors give other possible situations, such as a single vehicle passing, going rightward (violet). On the left-going lane the density is near critical for jams (0.1) and the density is low (0.01) on the right-going lane (500 cells, periodic boundary conditions). The route is divided into a passing zone (left half of the figure) and a no-passing zone (right half of the figure). Jams occur much more frequently in the no-passing zone. All vehicles have the same maximum velocity of 5.

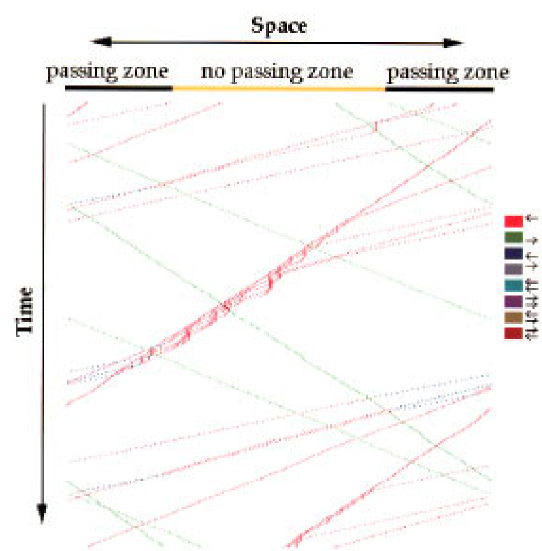

FIG. 4. (Color) Space-time diagram generated by a multispeed model. Each car has a maximum velocity, uniformly distributed between 2 and 5. In the center half of the figure no passing is allowed, while on the two sides passing is allowed. Slower vehicles trace steeper lines. Note in particular the large knot of cars near the center of the figure formed by a slow (left-going) car entering a no-passing zone. As the knot passes the leftmost boundary of the no-passing zone, it disperses.

are occupied; D_{limit}: the maximum local density for a safe pass.
Space1: true if ($\text{gap}_{\text{same}} < l_{\text{pass}}$) AND ($\text{gap}_{\text{opp}} > l_{\text{security}}$) AND ($\text{gap}_{\text{behind}} > l_{\text{back}}$).
Space2: true if ($\text{gap}_{\text{opp}} > l_{\text{security}}$) AND ($\text{gap}_{\text{behind}} > l_{\text{back}}$).
In our simulations we use the following values throughout: $l_{\text{pass}} = v$, $l_{\text{back}} = v_{\max}$, $l_{\text{security}} = 2 \times v_{\max} + 1$, $D_L = 2/l_{\text{density}}$, $p_{\text{change}} = 0.7$, and $p_{\text{decel}} = 0.5$.
Lane changes are determined by the following steps:

(1) IF [(H AND space1) AND ($D_L \leqslant D_{\text{limit}}$) AND (rand $< p_{\text{change}}$)] THEN change lane.
(2) IF [(not(H)) AND ($\text{gap}_{\text{same}} < l_{\text{security}}$) OR (Space2)] THEN change lane.

The first condition affects vehicles on their home lane. If a vehicle is in front of them, of the same sign, and at a distance less than l_{pass}, then they would like to pass that vehicle. However, a pass will only be initiated if there is room far enough ahead on the passing lane, and the number of cars in front of the vehicle it would like to pass is small. Passing occurs randomly, even if all these conditions are met, the probability of changing lanes is denoted p_{change}. The second condition concerns vehicles in the midst of passing. They return to their home lane if forced to by an oncoming vehicle, or if there is space enough on the home lane that they can return without braking.

Forward motion of a vehicle is determined as follows:

(1) IF [($|v| \neq = v_{\max}$)] THEN $v = v + \text{sgn}(v)$.
(2) IF [(oncoming) AND ($\text{gap}_{\text{same}} \leqslant (2 \times v_{\max} - 1)$)] THEN $v = \lfloor \text{gap}_{\text{same}}/2 \rfloor$.
(3) IF [(not(oncoming)) AND ($|v| > \text{gap}_{\text{same}}$)] THEN $v = \text{sgn}(v) \times \text{gap}_{\text{same}}$.
(4) IF [(H) AND ($|v| > 1$) AND (rand $< p_{\text{decel}}$) AND (not(oncoming))] THEN $v = v - \text{sgn}(v)$.
(5) IF [(H) AND (oncoming) AND ($|v| > 1$)] THEN $v = v - \text{sgn}(v)$.

These rules (1) accelerate the vehicle to maximum velocity, (2) rapidly decelerate the vehicle if there is an oncoming car too close, (3) decelerate the vehicle if it is closing in on another, both in their home lane, and (4) randomly decelerates the vehicle if it is on its home lane; if it is passing, it never decelerates randomly. Finally, (5) breaks the symmetry between the lanes, and thus prevents the emergence of a super jam.

IV. RESULTS

The usual way to represent the behavior of a CA is with a space-time diagram, where space is represented on the horizontal axis, and time on the vertical axis, with time proceeding downward. A typical space-time diagram for this model is shown in Fig. 2.

This figure illustrates that allowing passing makes traffic dramatically more fluid. The start-stop waves seen in the right half of the figure disappear in the left half. Here the densities have been chosen to maximize the effects caused by interactions between two lanes in opposite directions.

We have explored the entire range of densities on the two lanes, as shown in Fig. 3. Here the difference between the flow on the home lane in two-lane model with passing on both lanes is compared to the flow in a one-lane model (Δf). When the density on either or both lanes is large then there is little difference between the two-lane and one-lane models. When the density on the passing lane is small (< 0.1), then the flow on the home lane can be much greater than in a one-lane model (blue). Maximum improvement occurs near 0 density on the passing lane. If the density on the home lane is small (< 0.25) then the flow may be *lower* than in the corresponding one-lane model since when oncoming cars pass other oncoming cars they can impede traffic on the home lane. In an asymmetric model in which only vehicles on the home lane can pass, this slowing effect disappears. This experiment was performed with a value of p_{decel} of 0.5 and p_{change} of 0.5.

Our bidirectional model allows us to treat a host of new phenomena. For instance, the multispeed variant (Fig. 4) can produce a complicated knot of interactions resulting from a slow car leading a group of others through a no-passing zone. While in the no-passing zone the faster cars bunch up behind the slow car, forming a platoon. When the platoon reaches the end of the no-passing zone the platoon disperses.

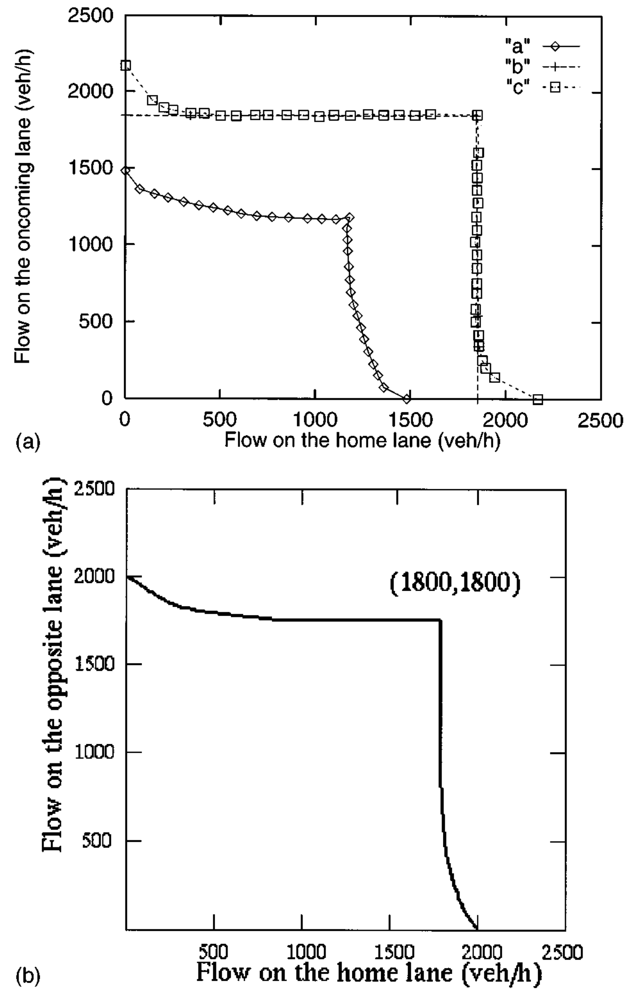

FIG. 5. Comparison of simulation results with experimental data. Left panel: The results of simulations under three conditions, (a) $p_{\text{decel}} = 0.5$, $p_{\text{change}} = 0.5$, (b) $p_{\text{decel}} = 0.25$, $p_{\text{change}} = 0$, and (c) $p_{\text{decel}} = 0.25$, $p_{\text{change}} = 0.3$. Right panel: corresponding physical measurements, modified from Fig. 5 of [5].

V. COMPARISON WITH EXPERIMENT

In Fig. 5, we compare simulation results (top panel) with the measurements of Yagar [5] (bottom panel). These experimental data are supported by traffic volume data from Australia and Canada [5]. Yagar measured the maximum flow that can be obtained on the home lane, as a function of the oncoming flow on the passing lane. We made comparable measurements on our simulations, with a variety of parameter settings, so as to approximate the parameter values implicit in the experimental data.

In the simulations, we obtain maximum total flow when vehicles are distributed uniformly, that is, half in each direction. The simulation curves are obtained by varying the total number of vehicles.

In both physical experiment and computer simulation, when densities are high, flows are not affected by passing, resulting in a corner in the graph of maximum flow. When one of the lane densities is fixed to a small enough value, the maximum flow increases on the other lane due to passing. Note that while our simulations are in close qualitative agreement with these experimental results of [5], these same results are *not* in agreement with the results reported in the previous [6] and current [7] editions of the influential Highway Capacity Manual (HCM). The previous HCM [6] presents a straight line of slope -1, while the current HCM [7] presents a smoother curve, which suggests that flows are correlated at high densities. Our simulation, built from close modeling of microscopic interactions, gives theoretical support for the view that the HCM's treatment of bidirectional traffic should be revised in favor of the results of Yagar.

VI. DISCUSSION

We have shown that complicated interactions between vehicles traveling on two-lane rural roads can be largely accounted for by a few simple rules. By small extension of previous work, we have greatly increased the realism that can be expected from cellular automaton models of vehicular traffic. Traffic in the two-lane bidirectional model approximates real traffic on both a microscopic and macroscopic level. Indeed, our results suggest revision of a standard text in highway studies.

Though systematic exploration of the parameter space should surely be undertaken, our model is robust to small changes in parameter value. Throughout the experiments described in this paper, we have fixed the values of most parameters at reasonable values, and focused on the two most important variables: the density of vehicles on each lane. We have localized the regions where lane interactions are strongest, and explored how these interactions occur. Due to the high relative speeds involved, these interactions have practical importance in traffic safety.

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